

Higgs Boson as a Dilaton

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Abstract

We study possible phenomenological consequences of the recently proposed new approach to the Weinberg-Salam model. The electroweak theory is considered as a gravity and the Higgs particle is interpreted in it as a dilaton, without the usual potential of interaction in the Higgs sector. We have taken as a test the process of photons pair production, $e^+ + e^- \rightarrow Z + \gamma + \gamma$. In the framework of new formulation this reaction is mediated in the lowest order by the dilaton. The cross section is found to be rather small.

1 Introduction

Recently, a new formulation of the Weinberg-Salam model has been suggested [1, 2, 3, 4] (the similar ideas are discussed also in papers [5, 6, 7]). A novel feature of this approach is the interpretation of the electroweak theory in terms of a gravity theory with the Higgs field as a dilaton.

The bosonic sector of the standard electroweak theory given by the Lagrangian including Yang-Mills triplet W_μ^a , abelian vector field Y_μ and complex scalar Higgs dublet $\Phi = (\phi_1, \phi_2)^1$,

$$\mathcal{L}_{WS} = -\frac{1}{4} G_{\mu\nu}^2(W) - \frac{1}{4} F_{\mu\nu}^2(Y) + |D_\mu \Phi|^2 - \mu^2 |\Phi|^2 - \lambda |\Phi|^4, \quad (1)$$

can be equivalently reformulated as an effective gravity²,

$$\mathcal{L}_{WS} = \sqrt{-\mathcal{G}} [-(M_p^2 R + \lambda) + \mathcal{L}_M]. \quad (2)$$

Here R is the scalar curvature, M_p plays the role of a Plank mass and the second term is the matter Lagrangian,

$$\begin{aligned} \mathcal{L}_M = & -\frac{1}{4} \mathcal{G}^{\mu\rho} \mathcal{G}^{\nu\sigma} G_{\mu\nu} G_{\rho\sigma} - \frac{1}{4} \mathcal{G}^{\mu\rho} \mathcal{G}^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} + \frac{1}{2} M_Z^2 \mathcal{G}^{\mu\nu} Z_\mu Z_\nu \\ & + M_W^2 \mathcal{G}^{\mu\nu} W_\mu^+ W_\nu^- \end{aligned} \quad (3)$$

¹Our notations are a bit different from those used in the papers [1] - [4]

²The Higgs mass $\mu = 0$ in \mathcal{L}_{WS} , the case $\mu \neq 0$ is discussed in the next section.

including massive vector bosons Z and W^\pm and massless photon A_μ . Lagrangian \mathcal{L}_M does not contain Higgs boson, which is interpreted in this approach as a dilaton and gives rise to the first, gravity, term. The metric tensor is always taken to be conformally flat,

$$\mathcal{G}_{\mu\nu} = \frac{\rho^2}{m^2} \eta_{\mu\nu}, \quad (4)$$

with flat Minkowski metric $\eta_{\mu\nu} = (+ - - -)$, where ρ is the modulus of the Higgs field, $\rho^2 = |\Phi|^2$, and m is the arbitrary parameter having dimension of mass. It provides a scale through which all other mass parameters are expressed,

$$\frac{1}{4} g^2 m^2 = M_W^2, \quad \frac{1}{2} (g^2 + g'^2) m^2 = M_Z^2, \quad M_p^2 = \frac{1}{6} m^2. \quad (5)$$

The kinetic part of the dilaton field, $(\partial\rho)^2$, turns into the scalar curvature in the gravity action³,

$$\frac{m^2}{6} \sqrt{-\mathcal{G}} R = -(\partial\rho)^2 + \text{total derivatives}.$$

The vector bosons acquire finite masses for $m \neq 0$ in the absence of the usual Higgs mechanism based on a symmetry breaking. There is no strict relation between the value of m and scalar potential parameter λ or Higgs mass μ , the non-zero masses can be generated in this interpretation even for $\lambda = 0$. Moreover, the dilaton, which replaces the Higgs particle in the effective gravity action (2), has always zero mass. These properties mark an essential difference to conventional approaches that underly the Standard Model. This is the reason to study possible phenomenological consequences following from this theory.

2 Comments on perturbation theory

Before going to phenomenological aspects we should comment a little on the perturbation theory since we are about to exploit it for the effective gravity.

³There is a possible ambiguity related to the analytical continuation from Euclidean space, where the theory is originally formulated, to Minkowski space. It could change the sign in front of the curvature term in the gravity action [1]. However this sign seems not to be especially important for our purposes.

To develop the perturbation theory we need first to derive a functional integral. The functional measure is originally given by the product of appropriate measures for all fields in the standard model (bosonic part),

$$D\mu = \prod_x d\Phi_1 d\Phi_1^* d\Phi_2 d\Phi_2^* dY_\mu dW_\mu^a.$$

After transition to new variables the volume of gauge group $d\Omega$ is separated out explicitly, $D\mu = d\Omega D\mu_{WS}$, leaving the gauge invariant part [2],

$$D\mu_{WS} = \prod_x \rho^3 d\rho dZ_\mu dW_\mu^+ dW_\mu^- dA_\mu,$$

where ρ is the dilaton field, which unlike ordinary fields enters the measure with local factor. The similar local factor occurs in the functional measure for pure gravity, making it reparametrization invariant (for dimension four)⁴,

$$D\mathcal{G} = \prod_x \prod_{\mu \leq \nu} \mathcal{G}^{\frac{5}{2}} d\mathcal{G}^{\mu\nu}, \quad \mathcal{G} = \det \mathcal{G}_{\mu\nu}.$$

To ensure the reparametrization invariance of the effective gravity in the presence of matter fields the measure should be of the form

$$D\mu_{eff} = D\mathcal{G} \prod_x \mathcal{G}^{-2} dZ_\mu dW_\mu^+ dW_\mu^- dA_\mu,$$

however the power of local factor differs from that in $D\mu_{WS}$. In addition, the reparametrization invariance is violated by the Higgs mass term in the action, $\mu^2 |\Phi|^2 = m^2 \mu^2 \mathcal{G}_{\nu\nu}/4$. This complicates the direct match between electroweak and gravity theories beyond classical level.

We try to establish the connection starting from the completely covariant functional integral,

$$Z = \int D\mu_{eff} e^{-S_{WS}(\mathcal{G}) - \frac{1}{\gamma} W^2(\mathcal{G})},$$

with the action S_{WS} corresponding to Lagrangian (2) without Higgs mass term. The square of Weyl tensor added to the action, $W^2(\mathcal{G}) = \sqrt{\mathcal{G}} W_{\mu\nu\lambda\sigma}^2$, enforces the metric to conformally flat form for $\gamma \rightarrow 0$ ($W_{\mu\nu\lambda\sigma} = 0$ for conformally flat metric)[4, 1]. One should stress the point that the limit $\gamma \rightarrow 0$ actually singles out the metrics, which can be brought to the form (4)

⁴Here the Euclidean signature is assumed for the metric tensor.

after coordinate transformation. To get the form (4) precisely the coordinates have to be fixed by four (for $D = 4$) gauge conditions,

$$\delta[\eta(\mathcal{G}^{\mu\nu}) - \ell] = \delta(\sqrt{\mathcal{G}}\mathcal{G}^{11} - \ell) \delta(\sqrt{\mathcal{G}}\mathcal{G}^{22} - \ell) \delta(\sqrt{\mathcal{G}}\mathcal{G}^{33} - \ell) \delta(\sqrt{\mathcal{G}}\mathcal{G}^{44} - \ell), \quad (6)$$

with $\ell(x) = \rho^2(x)/m^2$. After this the integral over $\mathcal{G}^{\mu\nu}$ turns actually into the integral over coordinate transformations preserving the metric $\mathcal{G}_{\mu\nu} = \ell\delta_{\mu\nu}$, so that

$$\begin{aligned} Z(\ell) &= \int D\mu_{eff} e^{-S_{WS}(\mathcal{G}) - \frac{1}{\gamma}W^2(\mathcal{G})} \delta[\eta(\mathcal{G}^{\mu\nu}) - \ell] \\ &= \frac{1}{\Delta(\ell)} \int dZ_\mu dW_\mu^+ dW_\mu^- dA_\mu e^{-S_{WS}(\ell)}. \end{aligned} \quad (7)$$

The function

$$\frac{1}{\Delta(\ell)} = \int D\mathcal{G} \prod_x \mathcal{G}^{-2} e^{-\frac{1}{\gamma}W^2(\mathcal{G})} \delta[\eta(\mathcal{G}^{\mu\nu}) - \ell]$$

involves the integral over residual for $\gamma \rightarrow 0$ coordinate transformations. Introducing invariant functional measure $D\Omega$ on the group of coordinate transformations, $\mathcal{G}^{\mu\nu} \rightarrow \mathcal{G}_\Omega^{\mu\nu}$, we can rewrite it as

$$\frac{1}{\Delta(\ell)} = \frac{1}{J(\ell)} \int D\Omega \delta[\eta(\mathcal{G}_\Omega^{\mu\nu}) - \ell] = \frac{1}{J(\ell)} \frac{1}{\Delta_{\text{FP}}(\ell)}, \quad (8)$$

where $\Delta_{\text{FP}}(\ell)$ is the standard Faddeev-Popov determinant for gravity, function $J(\ell)$ comes from Jacobian relating the measures $D\mathcal{G} \prod_x \mathcal{G}^{-2}$ and $D\Omega$.

Equation (7), (8) allows to connect the functional integrals for electroweak theory and gravity,

$$\begin{aligned} Z_{WS} &= \int D\mu_{WS} e^{-S_{WS}(\ell) - m^2\mu^2\ell} \\ &= \int D\ell \ell^{\frac{3}{2}} \Delta(\ell) e^{-m^2\mu^2\ell} \int D\mu_{eff} e^{-S_{WS}(\mathcal{G}) - \frac{1}{\gamma}W^2(\mathcal{G})} \delta[\eta(\mathcal{G}^{\mu\nu}) - \ell]. \end{aligned}$$

Here we denote through $m^2\mu^2\ell = \int d^4x \mu^2 |\Phi|^2$ the part of the action due to Higgs mass term, $\ell^{\frac{3}{2}} \sim \rho^3$ stands for the local factor in the integration measure $D\mu_{WS}$. Using the fact that $\Delta_{\text{FP}}(\mathcal{G}^{\mu\nu}) = \Delta_{\text{FP}}(\ell)$ in the integral over metrics we finally have ($\gamma \rightarrow 0$)

$$\begin{aligned} Z_{WS} &= \int D\ell J(\ell) \ell^{\frac{3}{2}} e^{-m^2\mu^2\ell} Z_{gr}, \\ Z_{gr} &= \int D\mu_{eff} e^{-S_{WS}(\mathcal{G}) - \frac{1}{\gamma}W^2(\mathcal{G})} \Delta(\mathcal{G}^{\mu\nu}) \delta[\eta(\mathcal{G}^{\mu\nu}) - \ell]. \end{aligned} \quad (9)$$

The Z_{gr} integral looks like a standard functional integral for gravity interacting with matter fields written in the particular gauge (6). According to general treatment Z_{gr} does not depend on the gauge fixing parameter ℓ (at least perturbatively), therefore the integral over ℓ with any weight functional results into general normalization only. Thus we can conclude that electroweak theory is equivalent at quantum (perturbative) level to a gravity theory taken in a special gauge. Furthermore, the Higgs mass term in the action along with the local factor in the measure $D\mu_{WS}$ and function $J(\ell)$ are interpreted as a particular weight functional ($\sim J(\ell)\ell^{\frac{3}{2}}e^{-m^2\mu^2\ell}$) in the chosen gauge.

Thus although in general case the Higgs mass term violates the complete covariance of gravity theory we can treat this violation as a choice of special gauge. Only after imposing this gauge and fixing very special form of the weight functional the effective gravity becomes equivalent to the electroweak theory.

There are two possible ways to address the electroweak/gravity connection. One way is to carry out calculations in the gravity under the gauge (6)). This way we may reproduce all the well known Standard Model results. On another hand it looks attractive to take the "gravity" seriously and to consider the electroweak theory in the form which is gauge (reparametrization) invariant.

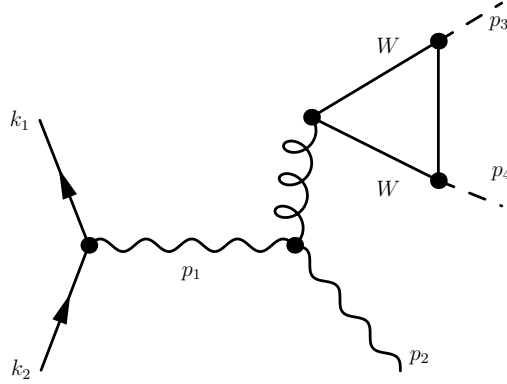
A suitable method is to deal with those quantities in the electroweak theory which are gauge (reparametrization) invariant with respect to the gravity. Any reasonable for the gravity gauge can be imposed in this case with output valid for the electroweak theory. Besides, the output value do not depend on the Higgs mass parameter μ^2 in the Lagrangian (1).

The last method can be applied to S -matrix (in Minkowski space), supposing its existence and reparametrization invariance in the gravity with asymptotically flat spacetime boundary conditions. It allows to find scattering amplitude directly in the effective gravity on the basis of conventional perturbation theory.

An interesting possibility caused by the universality of the gravitational interaction is the direct coupling of the Higgs-dilaton to the photon.

3 Photons pair production

We take the process of photons pair production off Z -boson, $e^+ + e^- \rightarrow Z \rightarrow Z + \gamma + \gamma$ shown in the Born order.



Photon pair production. p_1 and p_2 are Z -bosons momenta, curly line denotes the dilaton, $p_{3,4}$ are the final photons momenta.

To estimate the cross section of this reaction is interesting since:

- i) this process demonstrates the universality of our electroweak gravity
- ii) not excluded that the LEP2 data are already exclude (or confirm) this 'gravity-like' approach.

From the effective gravity viewpoint this process occurs in the lowest order through the virtual dilaton exchange. Although it looks rather simple there are two difficulties to deal with. First, the coupling of the gravity to the matter fields is universal and given by the energy-momentum tensor $\theta_{\mu\nu}$. In contrast to graviton the dilaton feels the trace part of $\theta_{\mu\nu}$ while the photon energy-momentum tensor is traceless. It makes scattering impossible at the Born level, the interaction should be mediated by loops.

Second problem comes from the fact that virtual Z -boson is produced in e^+e^- fusion, whereas the effective gravity describes pure bosonic sector only. To avoid this difficulty we treat the process using unitarity, which is assumed to hold for the whole amplitude. Having cut the diagram through virtual Z -boson line, we left with two mass shell amplitudes: $e^+e^- \rightarrow Z$ and $Z \rightarrow Z + \gamma + \gamma$. The former amplitude is really a standard electroweak neutral current vertex, while the latter amplitude has the form suitable to

apply the effective gravity⁵. After this the full amplitude is recovered by the dispersion relation with respect Z -boson invariant mass, which is simply amount to plugging the denominator of Z -boson propagator into obtained expression. This trick is similar to those frequently used in helicity based calculations [8].

We start first from the case, where $h \rightarrow \gamma + \gamma$ transition goes through W -boson loop. The dilaton vertices $ZZ \rightarrow h$, $h \rightarrow W^+W^-$, $W^+W^- \rightarrow \gamma$ as well as the massive W -boson propagator can be immediately read off from the matter Lagrangian (3). The loop has been calculated by standard dispersion relation taking the discontinuity with respect to the photon pair invariant energy, $S_{\gamma\gamma} = (p_3 + p_4)^2$ and subtraction point at $S_{\gamma\gamma} = 0$. The latter choice is dictated by the absence of real dilaton and photons interaction.

The dilaton propagator is extracted from the quadratic part of the gravity action,

$$\mathcal{L}_{gr} = \sqrt{-\mathcal{G}}[-M_p^2 R + \frac{1}{\gamma} W_{\mu\nu\lambda\sigma}^2],$$

with the second term enforcing the metric to conformally flat form for $\gamma \rightarrow 0$. Taking the metric as

$$g^{\mu\nu} = \eta^{\mu\nu} + \frac{1}{M_p^2} h^{\mu\nu}$$

and adding to the action a gauge-fixing term,

$$\begin{aligned} \mathcal{L}_{gf} = & -\frac{1}{4}\alpha h^{\mu\nu}[\eta_{\lambda\sigma}\eta_{\mu\nu}\partial^2 + \eta_{\nu\sigma}\partial_\lambda\partial_\mu + \eta_{\mu\sigma}\partial_\lambda\partial_\nu - 2\eta_{\mu\nu}\partial_\lambda\partial_\sigma - 2\eta_{\lambda\sigma}\partial_\mu\partial_\nu \\ & + \eta_{\lambda\nu}\partial_\mu\partial_\sigma + \eta_{\lambda\mu}\partial_\nu\partial_\sigma] h^{\lambda\sigma} \end{aligned}$$

arranged to produce harmonic (de Donder) gauge,

$$\partial_\mu h^{\mu\nu} - \frac{1}{2}\partial_\nu h^{\mu\mu} = 0,$$

we arrive at the propagator ($\gamma = 0$)

$$\begin{aligned} G_{\mu\nu\lambda\sigma}(k) = & \frac{1}{6k^6\alpha}[k^4\eta_{\lambda\sigma}\eta_{\mu\nu}\alpha + 6k^2k_\lambda k_\mu\eta_{\nu\sigma} + 6k^2k_\lambda k_\nu\eta_{\mu\sigma} + 2k^2k_\lambda k_\sigma\eta_{\mu\nu}\alpha \\ & + 2k^2k_\mu k_\nu\eta_{\lambda\sigma}\alpha + 6k^2k_\mu k_\sigma\eta_{\lambda\nu} + 6k^2k_\nu k_\sigma\eta_{\lambda\mu} + 4k_\lambda k_\mu k_\nu k_\sigma\alpha]. \end{aligned}$$

⁵In fact, we deal with kinematically allowed process $Z + Z \rightarrow \gamma + \gamma$ and use crossing symmetry to get the amplitude required.

One has to emphasize at this point that we assume the S -matrix element for $Z \rightarrow Z + \gamma + \gamma$ subprocess to be reparametrization invariant within effective gravity. It is this property that allows us to choose the convenient gauge and omit the Higgs mass term as has been discussed in the previous section.

Combining the dilaton exchange amplitude with the amplitude produced by the relevant piece of neutral current, where we neglect the lepton masses,

$$J_Z^\mu = \frac{g}{\cos \theta_W} [\bar{e} \gamma^\mu (\sin^2 \theta_W - \frac{1}{4}) e + \frac{1}{4} \bar{e} \gamma^\mu \gamma^5 e],$$

(θ_W is Weinberg angle), we get the imaginary part through which the final amplitude is restored. The cross section is summed up over two photons' polarizations, over three polarizations of outgoing Z -boson and averaged over incoming massless electron helicities. Finally it is integrated over phase volume with the invariant energy of the photon pair, $S_{\gamma\gamma} = (p_3 + p_4)^2$, fixed. The resulting one-loop cross section in the leading order both in $S_{\gamma\gamma}$ and the lepton pair invariant energy, $S = (p_1 + p_2)^2$, reads

$$\frac{d\sigma}{dS_{\gamma\gamma}} = \frac{g^2}{512\pi^3} \frac{1 - 4\sin^2 \theta_W + 8\sin^4 \theta_W}{\cos^2 \theta_W} \frac{\alpha_{em}^2}{16\pi^2} \frac{M_Z^2}{1152M_p^4} \frac{1}{S}.$$

There is a way to take into account also the contribution of fermion loop mediating $h \rightarrow \gamma + \gamma$ transition. Indeed, the fermion loop contribution is rather universal in QED and given by the well-known anomaly of energy-momentum tensor,

$$\theta_\mu^\mu = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu}, \quad (10)$$

relating the trace value to the electrodynamic β -function. On the other hand, this part of energy-momentum tensor provides the vertex for dilaton-photon coupling. Putting it together with pure boson loop found above we arrive at

$$\frac{d\sigma}{dS_{\gamma\gamma}} = \frac{g^2}{512\pi^3} \frac{1 - 4\sin^2 \theta_W + 8\sin^4 \theta_W}{\cos^2 \theta_W} \left[\frac{\alpha_{em}}{2\pi} - \frac{\beta(e)}{e} \right]^2 \frac{M_Z^2}{1152M_p^4} \frac{1}{S},$$

where $\beta(e) = n_f \frac{e^3}{12\pi^2}$ is one loop beta-function of QED with n_f different fermions.

Recalling that all parameters are expressed through a single scale in the effective gravity (5),

$$M_p^2 = \frac{1}{3} M_Z^2 \frac{\cos^2 \theta_W}{g^2},$$

we finally have for the one loop $e^+e^- \rightarrow Z + \gamma + \gamma$ cross section

$$\frac{d\sigma}{dS_{\gamma\gamma}} = \frac{9}{8} \frac{\alpha_{em}^5}{16\pi^2} \frac{1 - 4\sin^2\theta_W + 8\sin^4\theta_W}{\sin^6\theta_W \cos^6\theta_W} \left(1 - \frac{2}{3}n_f\right)^2 \frac{1}{1152M_Z^2} \frac{1}{S}. \quad (11)$$

In case of fermions with various charges, $e_f = e_Q e$, $1 - \frac{2}{3}n_f \rightarrow 1 - \frac{2}{3}\sum e_Q^2$.

Numerically the obtained cross section is extremely small, $\sigma \simeq 5 \cdot 10^{-9} \text{fb}$ for $n_f = 0$. Therefore we had no chance to observe the corresponding process at LEP2 where the integrated luminosity per experiment was of the order of 1fb^{-1} .

It could be of interest to compare above results with the cross section of the same process but with conventional massive scalar Higgs and standard electroweak vertices. The similar calculation with Higgs coupled to photons via W^\pm triangle gives for $S, S_{\gamma\gamma} \gg m_H^2$

$$\frac{d\sigma}{dS_{\gamma\gamma}} \simeq \frac{3}{64} \frac{\alpha_{em}^5}{16\pi^2} \frac{1 - 4\sin^2\theta_W + 8\sin^4\theta_W}{\sin^6\theta_W \cos^2\theta_W} \frac{M_Z^2}{SS_{\gamma\gamma}^2} \log^4 \frac{S_{\gamma\gamma}}{M_W^2}. \quad (12)$$

One more possible contribution occurs when the process goes through fermion loop. Taking the light quarks with masses m_Q we get in the lowest in m_Q order an additional term to the cross section (12),

$$\frac{d\sigma_f}{dS_{\gamma\gamma}} \simeq \sum_Q \frac{1}{32} \frac{\alpha_{em}^5}{16\pi^2} \frac{1 - 4\sin^2\theta_W + 8\sin^4\theta_W}{\sin^6\theta_W \cos^2\theta_W} \frac{m_Q^2 e_Q^2}{SS_{\gamma\gamma}^2} \log^2 \frac{S_{\gamma\gamma}}{M_W^2} \log^2 \frac{S_{\gamma\gamma}}{m_Q^2}. \quad (13)$$

Actually, it results from interference between W^\pm and quark triangles in the amplitude squared, the sum is taken over different fermions contributions whose masses and charges are m_Q and e_Q , the quark loop being proportional to m_Q with an extra m_Q appearing due to standard fermion-Higgs vertex.

The cross section has the same order in this case, though it is lesser than what follows from Eq.(11) for $n_f = 0$, the W^\pm triangle, Eq.(12), yields $\sigma \simeq 1.7 \cdot 10^{-9} \text{fb}$ for $\sqrt{S} = 14 \text{TeV}$. The reason for the discrepancy comes from the different asymptotic of the results for dilaton and conventional Higgs. According to the expressions (12), (13) the total cross section $e^+ + e^- \rightarrow Z + \gamma + \gamma$ decreases with the invariant energy, $\sigma \sim 1/S$. On the contrary, the expression (11) leads to the constant cross section, which, therefore, exceed the value predicted by Eqs.(12), (13) for large enough S . There is an additional difference between the effective gravity and standard model expressions. The term including fermion loop (13) vanishes for zero fermion

mass while the cross section evaluated through the dilaton exchange is finite for massless fermions, depending on their number n_f only.

The cross sections can be compared for the small energies too. Near threshold of the reaction, $\delta S = S - (M_Z + M_{\gamma\gamma})^2 \rightarrow 0$, assuming $S_{\gamma\gamma} = M_{\gamma\gamma}^2 \ll M_Z^2$, the dilaton exchange process yields in the lowest order in $M_{\gamma\gamma}/M_Z$

$$\begin{aligned} \frac{d\sigma}{dS_{\gamma\gamma}} &\simeq \frac{1}{2048} \frac{\alpha_{em}^5}{16\pi^2} \frac{1 - 4\sin^2\theta_W + 8\sin^4\theta_W}{\sin^6\theta_W \cos^6\theta_W} \frac{1}{S} \frac{S^2 - 2M_Z^4}{(S - M_Z^2)^2} \\ &\times \frac{1}{M_Z^3} \sqrt{\frac{M_{\gamma\gamma}}{M_Z}} \delta S \left(1 + \frac{8}{3} \sum e_Q^2\right)^2, \end{aligned} \quad (14)$$

(the opposite sign with respect to the high energy limit (11) in front of e_Q^2 in the last bracket is due to the W^\pm triangle amplitude behavior for small $S_{\gamma\gamma}$). It gives for the total cross section $\sigma \sim 10^{-9}$ fb for $\sqrt{S} = 200$ GeV and for $n_f = 0$.

The cross section value for the standard massive Higgs attached to the W^\pm triangle is estimated for $S_{\gamma\gamma} \ll M_Z^2 \ll m_H^2$ as

$$\begin{aligned} \frac{d\sigma}{dS_{\gamma\gamma}} &\simeq \frac{25}{64} \frac{\alpha_{em}^5}{16\pi^2} \frac{1 - 4\sin^2\theta_W + 8\sin^4\theta_W}{\sin^6\theta_W \cos^2\theta_W} \frac{1}{S} \frac{1}{(S - M_Z^2)^2} \frac{M_Z^5 M_{\gamma\gamma}^4}{m_H^4 M_W^4} \\ &\times \sqrt{\frac{M_{\gamma\gamma}}{M_Z}} \delta S, \end{aligned} \quad (15)$$

Like the previous expression (14), this formula is derived in the lowest order in $M_{\gamma\gamma}/M_Z$. Although equations (14) and (15) look different, their numeric values are rather close. Taking $\sqrt{S} = 200$ GeV and $m_H = 200$ GeV we get for Eq. (15) $\sigma \simeq 10^{-9}$ fb.

Thus, summarizing the above results and limiting cases, the cross sections due to dilaton and standard massive Higgs exchange are almost the same near threshold of the $e^+ + e^- \rightarrow Z + \gamma + \gamma$ reaction. The standard Higgs mediated cross section grows more rapidly with the energy and exceeds the dilaton one, which tends to a constant at the energies greater than the typical mass scale of the effective gravity, $S \gg M_p^2 \sim 5 \text{ TeV}^2$. However starting from the energies $\sqrt{S} \simeq 100$ GeV the Higgs exchange cross section decreases, becomes equal to the dilaton cross section at $\sqrt{S} \simeq 8 \text{ TeV}$ and goes to zero for asymptotically large energies.

4 Conclusion

There are two main ingredients in the electroweak theory treatment in terms of the effective gravity. The masses of vector bosons are introduced without appealing to symmetry breaking mechanism through Higgs field condensate. The Higgs particle is treated as massless dilaton. The term with Higgs mass in the original electroweak Lagrangian violates the reparametrization invariance of the effective gravity. However this term can be interpreted as a part of gauge-fixing functional for a special gauge chosen in the gravity. If the quantities we are interested in the electroweak theory are gauge invariant from the viewpoint of effective gravity (do not depend on the coordinate system), the perturbative relations (9) allow to obtain them directly in the effective gravity taking any convenient gauge.

Note that though the Higgs mass term μ^2 in the Lagrangian (1) does not affect the final result it could in principle depend on the Higgs self-interaction term λ , playing in gravity the role of cosmological constant. However S -matrix becomes subtle since the gravity is no more asymptotically flat in this case.

Recall that from the experimental viewpoint at the moment we know nothing about the self-interaction of Higgs boson. The conventional $\lambda|\Phi|^4$ potential is just the simplest possibility. Thus we may discuss another forms of the Higgs boson potential.

The one loop cross section $e^+ + e^- \rightarrow Z + \gamma + \gamma$ obtained in the effective gravity though very small shows the quantitative and qualitative difference with the cross section found in the standard electroweak theory. The discrepancy seems to come about due to asymptotically flatness of the effective gravity we have dealt with, that is $\lambda = 0$, which is not the case for the standard theory. Another reason probably lies in the fact that employing effective gravity approach we thereby reformulate in somewhat manner the perturbation theory, so that there is no order by order correspondence with those used in the standard model.

Strictly speaking we have to worry about the radiative corrections. The global fit to electroweak data in the standard Weinberg-Salam form indicates that the mass of the Higgs boson should be of the order of 100 GeV. In the case of a massless Higgs-dilaton the infrared behavior should be sensitive to space-time structure at infinity. The corresponding effect arises from the "curvature of the space" resulting from scalar field self-interaction. That is why it could be of interest to get further insight into effective gravity

approach, in particular, to study high orders of perturbative expansion.

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